F.1 MODEL FORMULATION

F.1.1 Unconstrained demand

\[ \frac{V_N}{V_0} = \left( \frac{P_N}{P_0} \right)^{E_P} \left[ FC_N CC_N / FC_0 \right]^{E_C} \left[ \frac{GDP_N}{GDP_0} \right]^{E_{GDP}} \]  

(1)

Where:

- \( \theta \) refers to the base year of 2008
- \( N \) refers to a future year (e.g. 1 = 2009, 2 = 2010 etc.)
- \( V \) = transport demand
- \( P \) = population and \( E_P \) the elasticity of demand with respect to population
- \( FC \) = fuel cost and \( CC \) the congestion charge multiplier and \( E_C \) the elasticity of demand with respect this combined cost (note \( CC_0 = 1 \) so not shown).
- \( GDP \) = GDP and \( E_{GDP} \) the elasticity of demand with respect to GDP

The elasticity values that were assumed were given in Table 1 (overleaf).

Road transport

The Department for Transport Long Distance Model (LDM) (Scott Wilson et al., 2007) seems to suggest an elasticity of journey time with respect to traffic of around 0.3, that is

\[ \frac{JT_N}{JT_0} = \left[ V_N / V_0 \right]^{0.3} \]  

(2)

However, increases in journey times will tend to inhibit demand: the LDM suggested an elasticity of \(-0.41\), that is

\[ \frac{V_N}{V_0} = \left[ \frac{JT_N}{JT_0} \right]^{-0.41} \right] \left[ V_N / V_0 \right] \]  

(3)

By iterating between the two formulae, (2) and (3), one obtains convergence to a stable solution, as illustrated in the worked example below:
### Table 1: Assumed elasticities

<table>
<thead>
<tr>
<th></th>
<th>Passenger demand</th>
<th>Freight demand</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Car</td>
<td>Rail</td>
</tr>
<tr>
<td><strong>Population</strong></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>‘Fuel price’</td>
<td>−0.2151</td>
<td>0.12</td>
</tr>
<tr>
<td><strong>GDP</strong></td>
<td>0.633</td>
<td>0.553</td>
</tr>
</tbody>
</table>

**Notes:**
- ‘Fuel price’ should perhaps more accurately be described here as ‘running cost’, as it includes a congestion charge multiplier, where applicable, however other motoring costs such as purchase and maintenance costs and other taxes are not included here.
- 1 National Transport Model (NTM) mean of high and low growth first round own price elasticities
- 2 NTM rail trip cross elasticity with respect to car fuel costs
- 3 NTM, GDP growth per capita
- 4 Balcombe et al. (2004). Table 9.23, public transport kms with respect to fuel price, long run.
- 5 Balcombe et al. (2004) (page 122). Mid-point between Dargay and Hanly’s range of −0.45 to −0.8
- 6 Oum et al. (1992) gave a mean price elasticity of −1.62 from 29 studies; Cowie (2010) suggested that fuel price represented around 20% of (legacy) airline’s costs, so we assumed a fuel price elasticity of −0.324.
- 7 Inter VISTAS study for IATA (2007)
- 8 De Jong et al. (2010)
- 9 1984 National Road Traffic Forecasts (according to McKinnon, 2006). It is also similar to a value produced by Shen et al. (2009), although this was for road and rail freight tonne kilometres combined.
- 10 Australian Bureau of Infrastructure, Transport and Regional Economics (BITRE), Transport Elasticities Database (Tables 9C01 (Rail), 9B15 (Road), 9D12 (Sea)). http://www.bitre.gov.au/tedb/
- 11 Set to be the same as road freight. Australia’s Bureau of Infrastructure, Transport and Regional Economics suggests a figure of −0.83 for short sea shipping with respect to price, however, fuel costs are only a small proportion of total costs.
- 12 Derivation from first principles based on the relationship
  - Cross elasticity of rail freight demand with respect to road price (0.225) =
  - Own elasticity of road demand with respect to road price (−0.1) x
  - Proportion of new rail demand that comes from road (−0.3) x
  - Ratio of road demand to rail demand (7.5) (from Transport Statistics Great Britain (DfT, 2010))
  - (Note: the −0.3 value was estimated due to lack of data)

---

**Worked example**

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Calculation</th>
</tr>
</thead>
</table>
| 1         | $V_N / V_0 = 2$  
Let’s assume that unconstrained demand is estimated to double by year $N$ |
| 2         | $JT_N / JT_0 = [V_N / V_0]^{0.3} = 1.23$  
Journey times increase by 23% |
| 3         | $V_N / V_0 = [JT_N / JT_0]^{-0.41} * V_N / V_0 = 0.92^{*2} = 1.84$  
Demand now only increased by 84% |
| 4         | $JT_N / JT_0 = [V_N / V_0]^{0.3} = 1.20$  
Journey times now only increased by 20% |
| 5         | $V_N / V_0 = [JT_N / JT_0]^{-0.41} * V_N / V_0 = 0.93^{*2} = 1.86$  
Demand now increased by 86% (compared with unconstrained demand) |
Repeating this procedure soon gives convergence to a journey time increase of 20.3% and a demand increase of 85.3%.

Note: the above procedure has been adjusted to take future year capacity into account.

**Rail transport**

The relationship between train delays and capacity utilisation (of the network) is of the form:

\[ \text{delay} = A \exp(\beta \text{ capacity utilisation}) \]  

where \( A \) and \( \beta \) are constants, with \( \beta \) typically having a value of around 2 (Faber Maunsell, 2007). Capacity utilisation is based on the ratio of actual train km per track km to maximum capacity, with a typical existing utilisation of around 50%.

So a change in delays, \( \frac{D_N}{D_0} \), is given by:

\[ \frac{D_N}{D_0} = \left[ \frac{\exp (\beta U_N)}{\exp (\beta U_0)} \right] \]  

However, increases in delays will tend to inhibit demand and capacity utilisation, with an elasticity of demand with respect to delays of \(-0.34\) (Preston and Dargay, 2005). That is,

\[ \frac{U_N}{U_0} = \left[ \frac{D_N}{D_0} \right]^{-0.34} \frac{U_N}{U_0} \]  

An iterative procedure, similar to that used for road transport above, was used to try to obtain convergence to the constrained demand. For example, if unconstrained capacity utilisation doubles, then the unconstrained delay is modelled to increase by a factor of 2.718 while the constrained increases are 1.619 for the capacity utilisation and 1.861 for the delay.

<table>
<thead>
<tr>
<th>Table 2: Constrained rail demand – examples with ( U_0 = 50% )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unconstrained capacity utilisation ratio ( U_N/U_0 )</strong></td>
</tr>
<tr>
<td><strong>Corresponding delay ratio ( D_N/D_0 )</strong></td>
</tr>
<tr>
<td><strong>Constrained capacity utilisation ratio ( U_N/U_0 )</strong></td>
</tr>
<tr>
<td><strong>Corresponding delay ratio ( D_N/D_0 )</strong></td>
</tr>
</tbody>
</table>

Note: convergence became more difficult to achieve as \( U_N/U_0 \) increased. For example, with \( U_N/U_0 = 3 \), 18 iterations were needed. And with \( U_N/U_0 = 5 \), convergence (to two decimal places) had not been obtained after 60 iterations. As the modelled unconstrained rail demand growth became as high as a factor of 6.5 (for the year 2083), this gave problems in calculating a constrained value in Excel using this iterative approach. The approach taken here was to use the formula derived from above using eight iterations where \( U_N/U_0 < 4 \) and to use:
Constrained value = $-68\% \times \text{unconstrained value for } \frac{U_U}{U_0} > 4$ \hfill (7)

where the percentage ratio was calculated from the last year before $\frac{U_U}{U_0} > 4$.

**F.2 PREVIOUS QUANTIFIED ASSESSMENTS**

Various reports and papers have been drawn upon to suggest possible transport infrastructure futures. They are summarised here organised by the topic area (highlighted in bold).

In terms of vehicle technologies, the main sources used were the TOSCA project (various reports found at www.toscaproject.org) and the DEcc 2050 Pathways Analysis (DECC, 2010). These sources suggested various, widely varying, levels of take-up of electric road vehicles, hybrid vehicles, use of hydrogenated vegetable oil as an alternative fuel for road freight, and rail electrification. For example, the DEcc 2050 pathways analysis considered four different levels of car/van technology uptake by the year 2050, with the use of the internal combustion engine in 2050 ranging from 77.5% to 0% for the different levels, reflecting the fact that the future in this area is highly uncertain.

The Tosca reports also provided data on the expected impacts of the vehicle technologies in terms of fuel consumption and CO$_2$ emissions. For example, they stated that the use of Hvo would reduce CO$_2$ emissions by more than 50% and would require only modest modifications to existing engines. These fuel consumption and emissions data were used in our models.

Our assumed capacity growth rates for the trunk road network were based on a paper by Glaister (2010). The existing growth rate, stated by Glaister, of about 100 lane km per annum, was taken as our ‘medium growth’ figure. ‘Low capacity growth’ is taken to be no growth from current levels. ‘High capacity growth’ was defined as an annual percentage increase of 0.5%, giving 53829 lane km by 2050.

Our assumed capacity growth rates for the rail network were based on planned schemes (Crossrail, HS2 and HS2+). The base year (2008) figure for the entire rail network is 32,160 track kilometres. ‘Low capacity growth’ assumes that this figure will remain constant, apart from an estimated increase of 237 km with the introduction of Crossrail in 2018 (www.crossrail.co.uk). ‘Medium capacity growth’ assumes that HS2 and HS2+ will go ahead as well but no other increases, giving:

- an estimated 410 km with the proposed introduction of HS2 by 2026 (www.hs2.org.uk)
- an estimated 1072 km with the proposed addition of HS2+ by 2035

‘High capacity growth’ assumes that all these planned schemes will go ahead, with an additional 0.2% per annum growth, in terms of track km, after 2035.

Previous studies of the historic relationships between transport demand and its drivers (population, energy costs, GDP etc.) have led to various elasticity values being formulated. The elasticities that were assumed for both passenger and freight demand are shown in Table 1. The elasticity with respect to population was assumed to be 1 across all transport modes, that is, a one-to-one relationship (e.g. population doubles, demand doubles). The sources used for the other elasticity figures are given below the table.

The Department for Transport estimated future growth in distance travelled, based on National Transport Model runs (Table 3).
The Department for Transport currently provides journey time and delay statistics (along with flow and speed) for the slowest 10% of routes on the trunk road network (DfT reference table no: cGN0202). This equates to data for 95 routes x 2 directions of travel. For example, using the year ending December 2008 data, the mean journey time per 10 miles, taken over the 190 measurements, was 13.58 min, of which 3.36 min (24.7%) was delay and 10.22 min was free flow journey time. These values were used for the base year data in the transport fast track analysis of delay.

Comparing our results with those of others, Mills et al. (2011) forecasted a road demand increase of 28% and a rail demand increase of 49% between 2010 and 2030, based on use of the National Transport Model. The comparable figures from the fast track analysis are:

- Roads – 22.6% (low growth) to 60.6% (high growth)
- Rail – 32.9% (low growth) to 69.9% (high growth)

In addition, Mills et al. (2011) estimated that the number of flights originating in the United Kingdom would increase by as much as 75% between 2010 and 2030. Our growth estimates for air during this period range from 60% (low) to 160% (high).
The Eddington Report (2006) derived its conclusions about future transport (up to 2025) from runs of the National Transport Model. Some of the assumptions made in the modelling are compared here with those used in the transport fast track analysis:

- GDP: The central estimate assumed 2.5% per annum growth (our central assumption was 2.3%).
- Fuel prices: The central estimate is that fuel prices are broadly flat, increasing by only 3% per annum. The fuel prices used in the fast track analysis were taken from DECC estimates; the central estimate saw fuel prices rise by 15% from 2008 to 2025, that is a rise of about 1% per annum.
- Fuel efficiency: The central estimate is that car fuel consumption falls by 23% or 1.2% annually; HGVs improve by 0.8% per year. These rates were greater than those used in the fast track analysis (taken from the Tosca project) which were 0.8% and 0.4% for cars and lorries, respectively.
- In addition, their modelling considered the value of time and working from home, neither of which have been considered in the fast track analysis.
REFERENCES


